

On the long-range gravity in warped backgrounds

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Abstract

In this paper the Randall-Sundrum model with brane-localized curvature terms is considered. Within some range of parameters a compact extra dimension in this model can be astronomically large. In this case the model predicts small deviation from Newton's law at astronomical scales, caused by the massive modes. The existence of this deviation can result in a slight affection on the planetary motion trajectories.

Keywords: Branes, induced gravity, long-range gravity

1 Introduction

Nowadays models with brane-localized curvature terms are widely discussed in the literature. In paper [1] it was argued that matter on the brane can induce a brane-localized curvature term via quantum corrections, which appears in the low-energy effective action. An attractive feature of this model is a modification of gravity at ultra-large scales, which can be very interesting from the cosmological point of view. Unfortunately a strong coupling effect was found in the model [2, 3]. Nevertheless, the DGP-proposal can be utilized not only in the models with infinite extra dimension. For example, in [2, 4, 5] some models with compact extra dimension and brane-localized curvature terms were discussed. The model proposed in [5] possesses some very interesting feature - an extra dimension in this model can be astronomically large. In the present paper we discuss another model with astronomically large extra dimension. It is based on the Randall-Sundrum solution for the metric [6] and was discussed in [7] in the case of sub-millimeter extra dimension. Here we consider this model from another point of view - in the case of a very large size of extra dimension.

2 The setup

The action of the model considered in [7] has the following form

$$S = S_g + S_1 + S_2 + S_{ind}, \quad (1)$$

where S_g , S_1 , S_2 and S_{ind} are given by

$$\begin{aligned} S_g &= \frac{1}{16\pi\hat{G}} \int (\hat{R} - \Lambda) \sqrt{-g} d^4x dy, \\ S_1 &= -\frac{1}{k16\pi\hat{G}} \int (\tilde{R} - \Lambda) \delta(y) \sqrt{-\tilde{g}} d^4x dy, \\ S_2 &= \frac{1}{k16\pi\hat{G}} \int (\tilde{R} - \Lambda) \delta(y - R) \sqrt{-\tilde{g}} d^4x dy, \\ S_{ind} &= \frac{\Omega_{ind}^2}{k16\pi\hat{G}} \int \tilde{R} \delta(y) \sqrt{-\tilde{g}} d^4x dy, \end{aligned} \quad (2)$$

Ω_{ind} is a dimensionless parameter and $\Lambda = -12k^2$. Here $\tilde{g}_{\mu\nu}$ is the induced metric on the branes and the subscripts 1 and 2 label the branes. The model possesses usual Z_2 orbifold symmetry. We also note that the signature of the metric g_{MN} is chosen to be $(-, +, +, +, +)$. Obviously, the model admits the Randall-Sundrum solution for the metric

$$ds^2 = \gamma_{MN} dx^M dx^N = \gamma_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (3)$$

where $\gamma_{\mu\nu} = e^{2\sigma(y)} \eta_{\mu\nu}$, $\eta_{\mu\nu}$ is the Minkowski metric and the function $\sigma(y) = -k|y|$ in the interval $-R \leq y \leq R$. The parameter k is positive and has the dimension of mass. The function σ has the properties

$$\partial_4 \sigma = -k \text{sign}(y), \quad \partial_4^2 \sigma = -2k(\delta(y) - \delta(y - R)) \equiv -2k\tilde{\delta}. \quad (4)$$

We denote $\hat{\kappa} = \sqrt{16\pi\hat{G}}$, where \hat{G} is the five-dimensional gravitational constant, and parameterize the metric g_{MN} as

$$g_{MN} = \gamma_{MN} + \hat{\kappa} h_{MN}, \quad (5)$$

h_{MN} being the metric fluctuations. In [7] the linearized equations of motion (for the field h_{MN}) corresponding to the action (1) were derived. It was shown, that in the gauge

$$h_{\mu 4} = 0, \quad h_{44} = h_{44}(x) \equiv \phi(x), \quad (6)$$

these *linearized* equations possess an additional symmetry under the transformations

$$h_{\mu\nu}(x, y) \rightarrow h_{\mu\nu}(x, y) + \sigma \gamma_{\mu\nu} \varphi(x) + \frac{1}{2k^2} \left(\sigma + \frac{1}{2} \right) \partial_\mu \partial_\nu \varphi(x), \quad (7)$$

$$\phi(x) \rightarrow \phi(x) + \varphi(x), \quad (8)$$

which *do not* belong to the gauge transformations. The validity of the gauge (6) was thoroughly checked in [7, 8, 9]. It is evident that with the help of transformations (7), (8) we can impose the condition $\phi(x) \equiv 0$. In other words it is necessary to examine the next order of perturbation theory to get the equation for the radion field. But since the radion field does not interact directly with matter on the brane, it seems that the linear approximation is not destroyed by the radion. The key feature of the model is that the only physically relevant case, in which tachyons and ghost are absent (and the symmetry (7), (8) is preserved) is when the matter (and the induced term S_{ind}) exists *on brane 1 only*. Brane 2 (at the point $y = R$) can be interpreted as a "naked" brane, i.e. a brane without matter on it (see [7]).

The gravity on the brane in the linear approximation is described by the field $h_{\mu\nu}$, which satisfies the following effective equation of motion [7]

$$\begin{aligned} & \frac{1}{2} \left(e^{-2\sigma} \square \left[h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right] + \partial_4^2 h_{\mu\nu} \right) - 2k^2 h_{\mu\nu} + 2k h_{\mu\nu} \tilde{\delta} - \\ & - \frac{1}{2k} \tilde{\delta} \left(e^{-2\sigma} \square \left[h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right] \right) + \frac{\Omega_{ind}^2}{2k} \delta(y) \left(e^{-2\sigma} \square \left[h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h \right] \right) = \\ & = -\frac{\hat{\kappa}}{2} \delta(y) t_{\mu\nu} - \frac{\hat{\kappa} k}{6\Omega_{ind}^2} \left(1 - \frac{\tilde{\delta}}{k} \right) \left[\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square} \right] t, \end{aligned} \quad (9)$$

$t_{\mu\nu}$ denoting the energy-momentum tensor of matter on the brane 1 and $t = \eta^{\mu\nu}t_{\mu\nu}$. Using this equation, we can get the equation for the zero mode of $h_{\mu\nu}$, which has the form $h_{\mu\nu}^0 = \alpha_{\mu\nu}e^{2\sigma}$. Let us multiply equation (9) by $e^{2\sigma}$ and integrate it over coordinate y . Using the orthonormality conditions, which have the form [7]

$$\int dy e^{-2\sigma} \left[1 - \frac{1}{k}\delta(y) + \frac{1}{k}\delta(y-R) + \frac{\Omega_{ind}^2}{k}\delta(y) \right] \Psi^l \Psi^n = \delta_{ln}, \quad (10)$$

we get

$$\square \left(\alpha_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\alpha \right) = -\frac{\hat{\kappa}k}{\Omega_{ind}^2} t_{\mu\nu}. \quad (11)$$

A fully analogous procedure was made in the case of RS1 model in [9].

In the next section we will estimate the effects produced by the massive modes. We will not solve equation (9), as it was made in [9] for the RS1 model, we will thoroughly estimate the masses of the modes, their wave functions and coupling constants to matter on the brane. We would like to remind that we will use other values of parameters, not that used in [7].

3 Wave functions of the massive modes

It was shown in [7], that equation for the wave functions of the massive modes has the form

$$\begin{aligned} & \frac{1}{2} (e^{-2\sigma} \square h_{\mu\nu} + \partial_4^2 h_{\mu\nu}) - 2k^2 h_{\mu\nu} + 2k\tilde{\delta} h_{\mu\nu} - \frac{1}{2k} \tilde{\delta} e^{-2\sigma} \square h_{\mu\nu} + \\ & + \frac{\Omega_{ind}^2}{2k} \delta(y) e^{-2\sigma} \square h_{\mu\nu} = 0. \end{aligned} \quad (12)$$

Following the footsteps of [7, 8], we arrive at the relations:

$$\Psi^n(y) = N_n \left(N_0 (t_n e^{kR}) J_2 (t_n e^{-\sigma}) - J_0 (t_n e^{kR}) N_2 (t_n e^{-\sigma}) \right), \quad (13)$$

where N_n is a normalization constant, and

$$\begin{aligned} & N_0 (t_n e^{kR}) J_0 (t_n) - J_0 (t_n e^{kR}) N_0 (t_n) + \\ & + \Omega_{ind}^2 [N_0 (t_n e^{kR}) J_2 (t_n) - J_0 (t_n e^{kR}) N_2 (t_n)] = 0. \end{aligned} \quad (14)$$

Here $h_{\mu\nu}(x, y) = \sum \Psi^n(y) h_{\mu\nu}^n(x)$, $\Psi^n(y)$ is the wave function of the corresponding massive mode, $t_n = \frac{m_n}{k}$ and m_n is such that $\square h_{\mu\nu}^n(x) = m_n^2 h_{\mu\nu}^n(x)$. Equation (14) defines the mass spectrum of the theory.

Let us choose Ω_{ind} such that $\Omega_{ind} \gg 1$ (we will specify its value later) and $kR \approx 1$. For the relatively large values of $\frac{m}{k}$ one can use the expansions for the Bessel functions for large values of arguments (see, for example, [10]). In this approximation masses of the modes are defined by

$$(e^{kR} - 1) \frac{m_n}{k} \approx \pi n, \quad (15)$$

where n is integer. Small corrections to these values are such that

$$\Psi^n(0) \sim N_n \frac{k^2}{m_n^2 \Omega_{ind}^2}, \quad (16)$$

This formula can be obtained from equation (14) (see Appendix 1). It seems that this form of the wave functions (inverse proportional to m_n^2) can be obtained only if there exist brane-localized curvature terms. It is necessary to note that since the main purpose of [7] was to examine the structure of linearized equations of motion, only a brief analysis of $\Psi^n(0)$, resulted in a very rough estimate $\Psi^n(0) \sim \frac{1}{\Omega_{ind}}$, was carried out in that paper. This estimate is sufficient for not rejecting the model in the case of sub-millimeter extra dimension. Equation (16) shows that contributions of the massive modes appear to be suppressed much stronger, and for the case of sub-millimeter extra dimension (an example in [7]) an effective theory on the brane actually describes massless 4-D gravity. At the same time the form of (16) is very important for the case of very large extra dimension, which will be considered below.

It is not difficult to show that integral in (10) is always positive and $N_n \sim \frac{m_n}{\sqrt{k}}$ for relatively large m_n (see Appendix 2). Thus the form of interaction of the massive modes with matter on the brane looks like

$$\frac{1}{2} \int_{brane} d^4x \left(\hat{\kappa} \sum_{n \leq j} \Psi^n(0) h_{\mu\nu}^n(x) t^{\mu\nu} + \frac{1}{M_{Pl} \Omega_{ind}} \sum_{n > j} \frac{a_n}{n} h_{\mu\nu}^n(x) t^{\mu\nu} \right), \quad (17)$$

where $a_n \sim 1$, $M_{Pl} = \Omega_{ind}/\sqrt{\hat{G}k}$ and j is such that for $n > j$ one can use expansion formulas for the Bessel functions with good accuracy. One can see, that coupling constants of the massive modes are suppressed by the factor Ω_{ind} and strongly depend on the number of a mode. It is not difficult to show that for $n \leq j$ the factor $\hat{\kappa}\Psi^n(0)$ is of the order of $1/(\Omega_{ind}M_{Pl})$. This estimate can be obtained from (10), (13) and (14) using the fact that the Bessel and Neumann functions are of the order of unity or lesser for the masses, which are the solutions of (14); and the fact that combination in the square brackets in (14) is equal to $\Psi^n(0)$ (accurate to the factor N_n).

Now let us discuss the possible values of parameters of the model. We would like to mention, that effective theory exactly of this type appears in the model considered in [5], so we will use some results obtained there. Because of the fact that coupling of Kaluza-Klein modes to matter on the brane is defined by the factors $(\Psi^n(0))^2$ and the sum $\sum \frac{1}{n^2}$ is rapidly convergent, one may not to worry about the strong affection of massive modes because of their quantity (it was thoroughly checked in [5]). This shows an importance of the fact that (16) is inverse proportional to m_n^2 and N_n is proportional to m_n . Let us consider R of the order, say, of the size of the Sun. For this case constraints on Ω_{ind} following from the astrophysical experiments are very severe (see [5]) - $\Omega_{ind} > 10^5$ (in our notations). The multi-dimensional Planck mass M should be of the order of 10 GeV to get the proper four-dimensional gravitational constant, which is defined by (11). Thus the hierarchy problem is solved in an analogous way to that proposed in [11, 12]. In this case the massive modes can become apparent on the distances of the order of extra dimension's size. It can lead to some slight affection on the Mercury motion.

Of course, the size of extra dimension can be much lesser: for example, from the size of the Earth down to the sub-millimeter scales and even smaller. In this case constraints on the value of Ω_{ind} following from the astrophysical data can be substantially relaxed, and the only claims are that the values of \hat{G} and Ω_{ind} with given R should give the proper value of four-dimensional gravitational constant and correspond to the constraints obtained from

the top-table gravitational experiments. Because of suppression of the massive modes, the phenomenology of this model differs considerably from that one of any standard "large extra dimensions" scenario (this issue was discussed in detail in [5]).

4 Discussion and final remarks

In this paper we constructed a self-consistent model with astronomically large compact extra dimension. One may ask: why we use warped geometry, whereas the same effect can be achieved in the model with flat background [5]? Firstly, in the paper [5] the radion field (44-component of the metric fluctuations) was taken into account not throughout all of calculations. It is well-known that existence of this field can change the theory considerably, as it was in the case of DGP-model [2, 3]. It is not probable that the same problem can arise in the model described in [5], nevertheless it is necessary to solve corresponding equations of motions with regard to the radion field more accurately, checking the validity of imposed gauge conditions. In our case considered above the radion field is absent in the linear approximation and one can forget about it (and about the problems with ghosts, see [2, 13, 14]) at least at the classical level. Secondly, the main advantage of warped geometry is that it ensures the proper tensor structure of the massless graviton propagator (in the case of flat geometry this property seems to be forbidden by the equation for the 44-component of metric fluctuations). It is also necessary to note, that the value of Ω_{ind} was chosen to correspond an appropriate value in [5]. More realistic minimal value, corresponding to the experimental data, can be a little bit larger (because of the warped background, see (17)). But it is evident that this difference should not be considerable. Some other interesting features of this setup were mentioned in [7].

Now let us discuss our choice for the size of extra dimension. Solution of equations of motion for the $\mu\nu$ -component of metric fluctuations includes some term (see [7]), which is a pure gauge from the four-dimensional point of view of an observer on the brane. This term is proportional to the factor $\frac{1}{k^2}$, and in the case of very large size of extra dimension (and correspondingly very small k) the linear approximation brakes down. Exactly the same thing happens in massive gravity at the small graviton mass limit, see for example [15]. Thus the size of extra dimension of the order of the Sun's size seems to be "safe" from this point of view. For a cases of much larger size of extra dimension it is necessary to carry out a more detailed analysis including examination of the non-linear effects.

We would like to mention, that only the case $e^{kR} \sim 1$ was considered in this paper. The case $e^{kR} \gg 1$ seems to be quite interesting, because there can appear an additional suppression of contributions of the massive modes by the exponential factor, which could relax the constraints on Ω_{ind} . At the same time the lowest modes will have lower masses $\sim ke^{-kR}$ in this case. Anyway this situation calls for more detailed investigation. Also it would be interesting whether the symmetry analogous to (7), (8) could arise in other models, for example, with infinite extra dimension, which admit modification of gravity at ultra-large scales.

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5 Appendix 1

Let us calculate estimates (15) and (16) for the case $\frac{m}{k} \gg 1$ and $kR \approx 1$. For large values of arguments we can use the following expansions for the Bessel functions [10]

$$J_0(t) \approx \sqrt{\frac{2}{\pi t}} \left[\cos\left(t - \frac{\pi}{4}\right) + \frac{1}{8t} \sin\left(t - \frac{\pi}{4}\right) \right], \quad (18)$$

$$N_0(t) \approx \sqrt{\frac{2}{\pi t}} \left[\sin\left(t - \frac{\pi}{4}\right) - \frac{1}{8t} \cos\left(t - \frac{\pi}{4}\right) \right], \quad (19)$$

$$J_2(t) \approx \sqrt{\frac{2}{\pi t}} \left[\cos\left(t - \frac{\pi}{4} - \pi\right) - \frac{15}{8t} \sin\left(t - \frac{\pi}{4} - \pi\right) \right], \quad (20)$$

$$N_2(t) \approx \sqrt{\frac{2}{\pi t}} \left[\sin\left(t - \frac{\pi}{4} - \pi\right) + \frac{15}{8t} \cos\left(t - \frac{\pi}{4} - \pi\right) \right]. \quad (21)$$

Substituting (18)-(21) into (14), we get

$$\begin{aligned} & \sin((\lambda - 1)t) + \left[\frac{1}{8t} - \frac{1}{8\lambda t} \right] \cos((\lambda - 1)t) = \\ & = -\Omega_{ind}^2 \left(\sin((\lambda - 1)t) - \left[\frac{15}{8t} + \frac{1}{8\lambda t} \right] \cos((\lambda - 1)t) \right), \end{aligned}$$

where $t = \frac{m}{k}$, $\lambda = e^{kR}$. For the relatively large t solutions of this equation in the zero approximation are defined by

$$t_n \approx \frac{\pi n}{\lambda - 1},$$

where n is integer. Corrections Δt_n to t_n at the first order are defined by the equation

$$(-1)^n \Delta t_n (\lambda - 1) (\Omega_{ind}^2 + 1) = (-1)^n \left[-\frac{1}{8t_n} + \frac{1}{8\lambda t_n} + \frac{\Omega_{ind}^2}{8\lambda t_n} + \frac{15\Omega_{ind}^2}{8t_n} \right],$$

from which one gets

$$\Delta t_n \approx \frac{1}{(\lambda - 1)t_n} \left[\frac{15}{8} + \frac{1}{8\lambda} - \frac{16}{8\Omega_{ind}^2} \right].$$

Finally for the function $\Psi^n(0)$ we have

$$\Psi^n(0) \approx N_n \sqrt{\frac{2}{\pi t_n}} \sqrt{\frac{2}{\pi t_n \lambda}} \left(\frac{16}{8\Omega_{ind}^2 t_n} \right) (-1)^n = \frac{4N_n}{\pi t_n^2 \Omega_{ind}^2 \sqrt{\lambda}} (-1)^n.$$

6 Appendix 2

Let us show, that

$$\int dy e^{-2\sigma} \left[1 - \frac{1}{k} \delta(y) + \frac{1}{k} \delta(y - R) + \frac{\Omega_{ind}^2}{k} \delta(y) \right] (\Psi)^2 > 0, \quad (22)$$

if Ψ satisfies the equation

$$\begin{aligned} & \frac{1}{2} (e^{-2\sigma} m^2 \Psi + \partial_4^2 \Psi) - 2k^2 \Psi + 2k \tilde{\delta} \Psi - \\ & - \frac{1}{2k} \tilde{\delta} e^{-2\sigma} m^2 \Psi + \frac{\Omega_{ind}^2}{2k} \delta(y) e^{-2\sigma} m^2 \Psi = 0. \end{aligned} \quad (23)$$

Here

$$\begin{aligned} \Psi(y) &= N Z_2(y), \\ Z_2(y) &= N_0 \left(\frac{m}{k} e^{kR} \right) J_2 \left(\frac{m}{k} e^{-\sigma} \right) - J_0 \left(\frac{m}{k} e^{kR} \right) N_2 \left(\frac{m}{k} e^{-\sigma} \right), \end{aligned}$$

N is normalization constant. It is easy to show that

$$\int_{-R}^R dy e^{-2\sigma} Z_2^2(y) = \left[t = \frac{m}{k} e^{-\sigma} \right] = \frac{2k}{m^2} \int_{t_0}^{t_R} t Z_2^2(t) dt, \quad (24)$$

where $t_0 = \frac{m}{k}$, $t_R = \frac{m}{k} e^{kR}$. With the help of the Bessel equation for the function $Z_2(t)$

$$\ddot{Z}_2(t) + \frac{\dot{Z}_2(t)}{t} + \left(1 - \frac{4}{t^2} \right) Z_2(t) = 0,$$

$\dot{Z}_2(t) = \frac{d}{dt} Z_2(t)$, one can get for the integral (24)

$$\begin{aligned} & \frac{2k}{m^2} \int_{t_0}^{t_R} Z_2(t) \left[\frac{4Z_2(t)}{t} - \dot{Z}_2(t) - t \ddot{Z}_2(t) \right] dt = \\ & = \frac{2k}{m^2} \int_{t_0}^{t_R} \left[t \dot{Z}_2^2(t) + \frac{4Z_2^2(t)}{t} \right] dt + \frac{2k}{m^2} \left(-t \dot{Z}_2(t) Z_2(t) \right) \Big|_{t_0}^{t_R}, \end{aligned}$$

where integration by parts were made. Using the boundary conditions for $Z_2(t)$, which follow from the equation (23) and have the form

$$\begin{aligned} & t \dot{Z}_2(t) + 2Z_2(t) - \frac{t^2}{2} Z_2(t) \Big|_{t=t_R} = 0, \\ & t \dot{Z}_2(t) + 2Z_2(t) - \frac{t^2}{2} Z_2(t) + \Omega_{ind}^2 \frac{t^2}{2} Z_2(t) \Big|_{t=t_0} = 0, \end{aligned}$$

we can show that integral (22) is equal to (accurate to the factor N^2)

$$\frac{2k}{m^2} \left(\int_{t_0}^{t_R} \left[t \dot{Z}_2^2(t) + \frac{4Z_2^2(t)}{t} \right] dt + 2 [Z_2^2(t)] \Big|_{t_0}^{t_R} \right). \quad (25)$$

Substituting the recurrent formula for the Bessel function

$$\dot{Z}_2(t) = Z_1(t) - \frac{2}{t}Z_2(t),$$

where

$$Z_1(t) = N_0 \left(\frac{m}{k} e^{kR} \right) J_1(t) - J_0 \left(\frac{m}{k} e^{kR} \right) N_1(t)$$

into (25), we get

$$\frac{2k}{m^2} \left(\int_{t_0}^{t_R} \left[t Z_1^2(t) - 4 \dot{Z}_2(t) Z_2(t) \right] dt + 2 \left[Z_2^2(t) \right]_{t_0}^{t_R} \right) = \frac{2k}{m^2} \int_{t_0}^{t_R} t Z_1^2(t) dt. \quad (26)$$

It is evident that this integral is always positive. Thus the integral in (22) is also positive for any real N .

Now let us calculate (26) for the relatively large m . Using the formulas analogous to (18)-(21) (see [10]) one can easily show that

$$Z_1(t) \approx \frac{2}{\pi \sqrt{t_R} \sqrt{t}} \cos(t - t_R).$$

Thus

$$\frac{2k}{m^2} \int_{t_0}^{t_R} t Z_1^2(t) dt \approx \frac{4k}{m^2 \pi^2} \frac{t_R - t_0}{t_R} = \frac{4k}{m^2 \pi^2} (1 - e^{-2kR})$$

and $N \sim \frac{m}{\sqrt{k}}$ for $kR \approx 1$.

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